S.S. Gabriyelyan

On characterized subgroups of Abelian topological groups X and the group of all X-valued null sequences

Comment.Math.Univ.Carolin. 55,1 (2014) 73 –99.

Abstract: Let X be an Abelian topological group. A subgroup H of X is characterized if there is a sequence $\mathbf{u} = \{u_n\}$ in the dual group of X such that $H = \{x \in X : (u_n, x) \to 1\}$. We reduce the study of characterized subgroups of X to the study of characterized subgroups of compact metrizable Abelian groups. Let $c_0(X)$ be the group of all X-valued null sequences and $\mathfrak{u}_0$ be the uniform topology on $c_0(X)$. If X is compact we prove that $c_0(X)$ is a characterized subgroup of $X^{\mathbb{N}}$ if and only if $X \cong \mathbb{T}^n \times F$, where $n \geq 0$ and F is a finite Abelian group. For every compact Abelian group X, the group $c_0(X)$ is a $\mathfrak{g}$-closed subgroup of $X^{\mathbb{N}}$. Some general properties of $(c_0(X), \mathfrak{u}_0)$ and its dual group are given. In particular, we describe compact subsets of $(c_0(X), \mathfrak{u}_0)$.

Keywords: group of null sequences; T-sequence; characterized subgroup; T-characterized subgroup; $\mathfrak{g}$-closed subgroup
AMS Subject Classification: Primary 22A10, 43A40; Secondary 54H11

References

[1] Arhangel'skii A.V., *Open and close-to-open mappings. Relations among spaces*, Trudy Moskov. Mat. Obsch. **15** (1966), 181–223.
[2] Arhangel'skii A.V., Tkachenko M.G., *Topological Groups and Related Structures*, Atlantis Press/World Scientific, Amsterdam-Paris, 2008.
[3] Armacost D., *The Structure of Locally Compact Abelian Groups*, Monographs and Textbooks in Pure and Applied Mathematics, 68, Marcel Dekker, Inc., New York, 1981.
[4] Aussenhofer L., *Contributions to the duality theory of Abelian topological groups and to the theory of nuclear groups*, Dissertation, Tübingen, 1998; Dissertationes Math. (Rozprawy Mat.) **384** (1999), 113p.
[5] Barbieri G., Dikranjan D., Milan C., Weber H., *Answer to Raczkowski's question on convergent sequences of integers*, Topology Appl. **132** (2003), 89–101.
[6] Barbieri G., Dikranjan D., Milan C., Weber H., *Convergent sequences in precompact group topologies*, Appl. Gen. Topol. **6** (2005), 149–169.
[7] Barbieri G., Dikranjan D., Milan C., Weber H., *Topological torsion related to some recursive sequences of integers*, Math. Nachr. **281** (2008), 930–950.
[8] Beiglböck M., Steineder C., Winkler R., *Sequences and filters of characters characterizing subgroups of compact Abelian groups*, Topology Appl. **153** (2006), 1682–1695.
[9] Biró A., *Strong characterizing sequences for subgroups of compact groups*, J. Number Theory **121** (2006), 324–354.
[10] Biró A., *Characterizations of groups generated by Kronecker sets*, J. Théor. Nombres Bordeaux **19** (2007), 567–582.
[11] Bíró A., Déshouillers J.-M., Sós V.T., *Good approximation and characterization of subgroups of* $\mathbb{R}/\mathbb{Z}$, Studia Sci. Math. Hungar. **38** (2001), 97–113.
[12] Biró A., Sós V.T., *Strong characterizing sequences in simultaneous Diophantine approximation*, J. Number Theory **99** (2003), 405–414.
[13] Borel J.P., *Sous-groupes de* $\mathbb{R}$ *liés à répartition modulo* 1 *de suites*, Ann. Fac. Sci. Toulouse Math. **5** (1983), 217–235.
[14] Chasco M.J., *Pontryagin duality for metrizable groups*, Arch. Math. **70** (1998), 22–28.
[15] Chasco M.J., Martín-Peinador E., Tarieladze V., *On Mackey topology for groups*, Studia Math. **132** (1999), 257–284.
[16] Comfort W.W., Raczkowski S.U., Trigos-Arrieta F.J., *Making group topologies with, and without, convergent sequences*, Appl. Gen. Topol. **7** (2006), 109–124.
[17] Comfort W.W., Ross K.A., *Topologies induced by groups of characters*, Fund. Math. **55** (1964), 283–291.

[18] Comfort W., Trigos-Arrieta F.-J., Ta Sun Wu, *The Bohr compactification, modulo a metrizable subgroup*, Fund. Math. **143** (1993), 119–136.

[19] Dikranjan D., Gabriyelyan S.S., *On characterized subgroups of compact abelian groups*, Topology Appl. **160** (2013), 2427–2442.

[20] Dikranjan D., Kunen K., *Characterizing subgroups of compact abelian groups*, J. Pure Appl. Algebra **208** (2007), 285–291.

[21] Dikranjan D., Martín-Peinador E., Tarieladze V., *Group valued null sequences and metrizable non-Mackey groups*, Forum Math. DOI: 10.1515/forum-2011-0099, February 2012.

[22] Dikranjan D., Milan C., Tonolo A., *A characterization of the MAP abelian groups*, J. Pure Appl. Algebra **197** (2005), 23–41.

[23] Dikranjan D., Prodanov I., Stojanov L., *Topological groups. Characters, Dualities, and Minimal Group Topologies*, Marcel Dekker, Inc., New York-Basel, 1990.

[24] Eggleston H.G., *Sets of fractional dimensions which occur in some problems of number theory*, Proc. London Math. Soc. **54** (1952), 42–93.

[25] Engelking R., *General Topology*, Panstwowe Wydawnictwo Naukowe, Waszawa, 1977.

[26] Gabriyelyan S.S., *Groups of quasi-invariance and the Pontryagin duality*, Topology Appl. **157** (2010), 2786–2802.

[27] Gabriyelyan S.S., *On T-sequences and characterized subgroups*, Topology Appl. **157** (2010), 2834–2843.

[28] Gabriyelyan S.S., *On a generalization of Abelian sequential groups*, Fund. Math. **221** (2013), 95–127.

[29] Gabriyelyan S.S., *Minimally almost periodic group topologies on infinite countable Abelian groups: A solution to a question by Comfort*, preprint, 2010, arXiv.org/abs/1002.1468.

[30] Gabriyelyan S.S., *On T-characterized subgroups of compact Abelian groups*, preprint, 2012.

[31] Glicksberg I., *Uniform boundedness for groups*, Canad. J. Math. **14** (1962), 269–276.

[32] Graev M., *Free topological groups*, Izv. Akad. Nauk SSSR Ser. Mat. **12** (1948), 278–324.

[33] Hart J.E., Kunen K,, *Limits in function spaces and compact groups*, Topology Appl. **151** (2005), 157–168.

[34] Hart J.E., Kunen K., *Limits in compact Abelian groups*, Topology Appl. **153** (2006), 991–1002.

[35] Hewitt E., Ross K.A., *Abstract Harmonic Analysis*, Vol. I, 2nd ed., Springer, Berlin, 1979.

[36] Kaplan S., *Extensions of the Pontryagin duality, I: Infinite products*, Duke Math. J. **15** (1948), 649–658.

[37] Kraaikamp C., Liardet P., *Good approximations and continued fractions*, Proc. Amer. Math. Soc. **112** (1991), 303–309.

[38] Larcher G., *A convergence problem connected with continued fractions*, Proc. Amer. Math. Soc. **103** (1988), 718–722.

[39] Nienhuys J., *Construction of group topologies on Abelian groups*, Fund. Math. **75** (1972), 101–116.

[40] Petersen K., Shapiro L., *Induced flows*, Trans. Amer. Math. Soc. **177** (1973), 375–390.

[41] Protasov I.V., Zelenyuk E.G., *Topologies on abelian groups*, Math. USSR Izv. **37** (1991), 445–460; Russian original: Izv. Akad. Nauk SSSR **54** (1990), 1090–1107.

[42] Raczkowski S.U., *Totally bounded topological group topologies on the integers*, Topology Appl. **121** (2002), 63–74.

[43] Rolewicz S., *Some remarks on monothetic groups*, Colloq. Math. **13** (1964), 28–29.

[44] Siwiec F., *Sequence-covering and countably bi-quotient mapping*, General Topology Appl. **1** (1971), 143–154.

[45] Winkler R., *Ergodic group rotations, Hartman sets and Kronecker sequences*, Monatsh. Math. **135** (2002), 333–343.