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Intersections of essential minimal prime ideals

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Abstract: Let $\mathcal{Z}(\mathcal{R})$ be the set of zero divisor elements of a commutative ring R with identity and $\mathcal{M}$ be the space of minimal prime ideals of R with Zariski topology. An ideal I of R is called strongly dense ideal or briefly sd-ideal if $I \subseteq \mathcal{Z}(\mathcal{R})$ and I is contained in no minimal prime ideal. We denote by $R_K(\mathcal{M})$, the set of all $a \in R$ for which $\overline{D(a)} = \overline{\mathcal{M} \setminus V(a)}$ is compact. We show that R has property (A) and $\mathcal{M}$ is compact if and only if R has no sd-ideal. It is proved that $R_K(\mathcal{M})$ is an essential ideal (resp., sd-ideal) if and only if $\mathcal{M}$ is an almost locally compact (resp., $\mathcal{M}$ is a locally compact non-compact) space. The intersection of essential minimal prime ideals of a reduced ring R need not be an essential ideal. We find an equivalent condition for which any (resp., any countable) intersection of essential minimal prime ideals of a reduced ring R is an essential ideal. Also it is proved that the intersection of essential minimal prime ideals of $C(X)$ is equal to the socle of C(X) (i.e., $C_F(X) = O^{\beta X \setminus I(X)}$). Finally, we show that a topological space X is pseudo-discrete if and only if $I(X) = X_L$ and $C_K(X)$ is a pure ideal.

Keywords: essential ideals; sd-ideal; almost locally compact space; nowhere dense; Zariski topology
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