Abdelouaheb Ardjouni, Ahcène Djoudi

Existence of periodic solutions for first-order totally nonlinear neutral differential equations with variable delay

Comment.Math.Univ.Carolin. 55,2 (2014) 215 –225.

Abstract: We use a modification of Krasnoselskii's fixed point theorem due to Burton (see [*Liapunov functionals, fixed points and stability by Krasnoselskii's theorem*, Nonlinear Stud. **9** (2002), 181–190], Theorem 3) to show that the totally nonlinear neutral differential equation with variable delay

$$x'(t) = -a(t)h(x(t)) + c(t)x'(t - g(t))Q'(x(t - g(t))) + G(t, x(t), x(t - g(t))),$$

has a periodic solution. We invert this equation to construct a fixed point mapping expressed as a sum of two mappings such that one is compact and the other is a large contraction. We show that the mapping fits very nicely for applying the modification of Krasnoselskii's theorem so that periodic solutions exist.

Keywords: periodic solution; nonlinear neutral differential equation; large contraction; integral equation
AMS Subject Classification: 34K20, 45J05, 45D05

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