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Semicontinuous integrands as jointly measurable maps

Abstract: Suppose that $(X, \mathcal{A})$ is a measurable space and Y is a metrizable, Souslin space. Let $\mathcal{A}^u$ denote the universal completion of $\mathcal{A}$. For $x \in X$, let $\underline{f}(x, \cdot)$ be the lower semicontinuous hull of $f(x, \cdot)$. If $f : X \times Y \to \overline{\mathbb{R}}$ is $(\mathcal{A}^u \otimes \mathcal{B}(Y), \mathcal{B}(\overline{\mathbb{R}}))$-measurable, then $\underline{f}$ is $(\mathcal{A}^u \otimes \mathcal{B}(Y), \mathcal{B}(\overline{\mathbb{R}}))$-measurable.

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