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*On some classes of spaces with the  $D$ -property*

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**Abstract:** We shall prove that under CH every regular meta-Lindelöf  $P$ -space which is locally  $D$  has the  $D$ -property. In addition, we shall prove that a regular submetalindelöf  $P$ -space is  $D$  if it is locally  $D$  and has locally extent at most  $\omega_1$ . Moreover, these results can be extended from the class of locally  $D$ -spaces to the wider class of  $\mathbb{D}$ -scattered spaces. Also, we shall give a direct proof (without using topological games) of the result shown by Peng [*On spaces which are  $D$ , linearly  $D$  and transitively  $D$* , Topology Appl. **157** (2010), 378–384] which states that every weak  $\bar{\theta}$ -refinable  $\mathbb{D}$ -scattered space is  $D$ .

**Keywords:** property  $D$ ; meta-Lindelöf; weak  $\bar{\theta}$ -refinable;  $P$ -space; scattered space

**AMS Subject Classification:** 54D20, 54A35, 54G10

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