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A symplectic representation of E_7

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Abstract: We explicitly construct a particular real form of the Lie algebra \mathfrak{e}_7 in terms of symplectic matrices over the octonions, thus justifying the identifications $\mathfrak{e}_7 \cong \mathfrak{sp}(6, \mathbb{O})$ and, at the group level, $E_7 \cong \mathrm{Sp}(6, \mathbb{O})$. Along the way, we provide a geometric description of the minimal representation of \mathfrak{e}_7 in terms of rank 3 objects called *cubies*.

Keywords: exceptional Lie algebras; octonions; E_7

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