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Mapping theorems on countable tightness and a question of F. Siwiec

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Abstract: In this paper ss-quotient maps and ssq-spaces are introduced. It is shown that (1) countable tightness is characterized by ss-quotient maps and quotient maps; (2) a space has countable tightness if and only if it is a countably bi-quotient image of a locally countable space, which gives an answer for a question posed by F. Siwiec in 1975; (3) ssq-spaces are characterized as the ss-quotient images of metric spaces; (4) assuming $2^\omega < 2^{\omega_1}$, a compact T_2-space is an ssq-space if and only if every countably compact subset is strongly sequentially closed, which improves some results about sequential spaces obtained by M. Ismail and P. Nyikos in 1980.

Keywords: countable tightness; strongly sequentially closed sets; sequentially closed sets; quotient maps; countably bi-quotient maps; locally countable spaces
AMS Subject Classification: 54B15, 54D55, 54E40

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