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Connectedness of some rings of quotients of $C(X)$ with the m-topology

Comment.Math.Univ.Carolin. 56,1 (2015) 63 –76.

Abstract: In this article we define the m-topology on some rings of quotients of $C(X)$. Using this, we equip the classical ring of quotients $q(X)$ of $C(X)$ with the m-topology and we show that $C(X)$ with the r-topology is in fact a subspace of $q(X)$ with the m-topology. Characterization of the components of rings of quotients of $C(X)$ is given and using this, it turns out that $q(X)$ with the m-topology is connected if and only if X is a pseudocompact almost P-space, if and only if $C(X)$ with r-topology is connected. We also observe that the maximal ring of quotients $Q(X)$ of $C(X)$ with the m-topology is connected if and only if X is finite. Finally for each point x, we introduce a natural ring of quotients of $C(X)/O_x$ which is connected with the m-topology.

Keywords: r-topology; m-topology; almost P-space; pseudocompact space; component; classical ring of quotients of $C(X)$
AMS Subject Classification: Primary 54C35; Secondary 54C40

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