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Todorcevic orderings as examples of ccc forcings without adding random reals

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Abstract: In [*Two examples of Borel partially ordered sets with the countable chain condition*, Proc. Amer. Math. Soc. **112** (1991), no. 4, 1125–1128], Todorcevic introduced a ccc forcing which is Borel definable in a separable metric space. In [*On Todorcevic orderings*, Fund. Math., to appear], Balcar, Pazák and Thümmel applied it to more general topological spaces and called such forcings *Todorcevic orderings*. There they analyze Todorcevic orderings quite deeply. A significant remark is that Thümmel solved the problem of Horn and Tarski by use of Todorcevic ordering [*The problem of Horn and Tarski*, Proc. Amer. Math. Soc. **142** (2014), no. 6, 1997–2000]. This paper supplements the analysis of Todorcevic orderings due to Balcar, Pazák and Thümmel in [*On Todorcevic orderings*, Fund. Math., to appear]. More precisely, it is proved that Todorcevic orderings add no random reals whenever they have the countable chain condition.

Keywords: Todorcevic orderings; random reals
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