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Symmetric products of the Euclidean spaces and the spheres

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Abstract: By $F_n(X)$, $n \geq 1$, we denote the n-th symmetric product of a metric space (X, d) as the space of the non-empty finite subsets of X with at most n elements endowed with the Hausdorff metric d_H. In this paper we shall describe that every isometry from the n-th symmetric product $F_n(X)$ into itself is induced by some isometry from X into itself, where X is either the Euclidean space or the sphere with the usual metrics. Moreover, we study the n-th symmetric product of the Euclidean space up to bi-Lipschitz equivalence and present that the 2nd symmetric product of the plane is bi-Lipschitz equivalent to the 4-dimensional Euclidean space.

Keywords: isometry; symmetric product; bi-Lipschitz maps; Euclidean space; sphere
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