Ioana Ghenciu

Property (**wL**) *and the reciprocal Dunford-Pettis property in projective tensor products*

Comment.Math.Univ.Carolin. 56,3 (2015) 319 –329.

Abstract: A Banach space X has the reciprocal Dunford-Pettis property ($RDPP$) if every completely continuous operator T from X to any Banach space Y is weakly compact. A Banach space X has the $RDPP$ (resp. property (wL)) if every L-subset of X^* is relatively weakly compact (resp. weakly precompact). We prove that the projective tensor product $X \otimes_\pi Y$ has property (wL) when X has the $RDPP$, Y has property (wL), and $L(X, Y^*) = K(X, Y^*)$.

Keywords: the reciprocal Dunford-Pettis property; property (wL); spaces of compact operators; weakly precompact sets
AMS Subject Classification: Primary 46B20, 46B28; Secondary 28B05

References

[1] Albiac F., Kalton N.J., *Topics in Banach Space Theory*, Graduate Texts in Mathematics, 233, Springer, New York, NY, USA, 2006.
[2] Ansari S.I., *On Banach spaces Y for which $B(C(\Omega), Y) = K(C(\Omega), Y)$*, Pacific J. Math. **169** (1995), 201–218.
[3] Bator E.M., *Remarks on completely continuous operators*, Bull. Polish Acad. Sci. Math. **37** (1989), 409–413.
[4] Bator E.M., Lewis P., *Operators having weakly precompact adjoints*, Math. Nachr. **157** (2006), 99–103.
[5] Bator E.M., Lewis P., Ochoa J., *Evaluation maps, restriction maps, and compactness*, Colloq. Math. **78** (1998), 1–17.
[6] Bessaga C., Pelczynski A., *On bases and unconditional convergence of series in Banach spaces*, Studia Math. **17** (1958), 151–174.
[7] Bombal F., *On V^* sets and Pelczynski's property V^**, Glasgow Math. J. **32** (1990), 109–120.
[8] Bombal F., Villanueva I., *On the Dunford-Pettis property of the tensor product of $C(K)$ spaces*, Proc. Amer. Math. Soc. **129** (2001), 1359–1363.
[9] Bourgain J., *New Classes of $\mathcal{L}_p$-spaces*, Lecture Notes in Math., 889, Springer, Berlin-New York, 1981.
[10] Bourgain J., *New Banach space properties of the disc algebra and H^∞*, Acta Math. **152** (1984), 1–2, 148.
[11] Bourgain J., *H^∞ is a Grothendieck space*, Studia Math. **75** (1983), 193–216.
[12] Delbaen F., *Weakly compact operators on the disk algebra*, J. Algebra **45** (1977), 284–294.
[13] Diestel J., *Sequences and Series in Banach Spaces*, Graduate Texts in Mathematics, 92, Springer, Berlin, 1984.
[14] Diestel J., *A survey of results related to the Dunford-Pettis property*, Contemporary Math. **2** (1980), 15–60.
[15] Diestel J., Uhl J.J., Jr., *Vector Measures*, Math. Surveys 15, American Mathematical Society, Providence, RI, 1977.
[16] Emmanuele G., *A remark on the containment of c_0 in spaces of compact operators*, Math. Proc. Cambr. Philos. Soc. **111** (1992), 331–335.
[17] Emmanuele G., *The reciprocal Dunford-Pettis property and projective tensor products*, Math. Proc. Cambridge Philos. Soc. **109** (1991), 161–166.
[18] Emmanuele G., *On the containment of c_0 in spaces of compact operators*, Bull. Sci. Math. **115** (1991), 177–184.
[19] Emmanuele G., *Dominated operators on $C[0,1]$ and the (CRP)*, Collect. Math. **41** (1990), 21–25.
[20] Emmanuele G., *A dual characterization of Banach spaces not containing ℓ^1*, Bull. Polish Acad. Sci. Math. **34** (1986), 155–160.
[21] Emmanuele G., John K., *Uncomplementability of spaces of compact operators in larger spaces of operators*, Czechoslovak Math. J. **47** (1997), 19–31.

[22] Ghenciu I., Lewis P., *The embeddability of c_0 in spaces of operators*, Bull. Polish. Acad. Sci. Math. **56** (2008), 239–256.
[23] Ghenciu I., Lewis P., *The Dunford-Pettis property, the Gelfand-Phillips property and (L)-sets"*, Colloq. Math. 106 (2006), 311–324.
[24] Ghenciu I., Lewis P., *Almost weakly compact operators*, Bull. Polish. Acad. Sci. Math. **54** (2006), 237–256.
[25] Grothendieck A., *Sur les applications linéaires faiblement compactes d'espaces du type $C(K)$*, Canad. J. Math. **5** (1953), 129–173.
[26] Kalton N., *Spaces of compact operators*, Math. Ann. **208** (1974), 267–278.
[27] Leavelle T., *Dissertation*, UNT.
[28] Lindenstrauss J., Tzafriri L., *Classical Banach Spaces II*, Springer, Berlin-New York, 1979.
[29] Pełczyński A., *On Banach spaces containing $L^1(\mu)$*, Studia Math. **30** (1968), 231–246.
[30] Pełczyński A., *Banach spaces on which every unconditionally converging operator is weakly compact*, Bull. Acad. Polon. Sci. Math. Astronom. Phys. **10** (1962), 641–648.
[31] Pełczyński A., Semadeni Z., *Spaces of continuous functions (III)*, Studia Math. **18** (1959), 211–222.
[32] Pisier G., *Factorization of Linear Operators and Geometry of Banach Spaces*, CBMS Regional Conf. Series in Math. 60, American Mathematical Society, Providence, RI, 1986.
[33] Pitt H.R., *A note on bilinear forms*, J. London Math. Soc. **11** (1936), 174–180.
[34] Ryan R.A., *Introduction to Tensor Products of Banach Spaces*, Springer, London, 2002.
[35] Saab E., Saab P., *On stability problems of some properties in Banach spaces*, in: K. Sarosz (ed.), Function Spaces, Lecture Notes Pure Appl. Math., 136, Dekker, New York 1992, 367–394.
[36] Tzafriri L., *Reflexivity in Banach lattices and their subspaces*, J. Functional Analysis **10** (1972), 1–18.