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Addition theorems for dense subspaces

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Abstract: We study topological spaces that can be represented as the union of a finite collection of dense metrizable subspaces. The assumption that the subspaces are dense in the union plays a crucial role below. In particular, Example 3.1 shows that a paracompact space X which is the union of two dense metrizable subspaces need not be a p-space. However, if a normal space X is the union of a finite family μ of dense subspaces each of which is metrizable by a complete metric, then X is also metrizable by a complete metric (Theorem 2.6). We also answer a question of M.V. Matveev by proving in the last section that if a Lindelöf space X is the union of a finite family μ of dense metrizable subspaces, then X is separable and metrizable.

Keywords: dense subspace; perfect space; Moore space; Čech-complete; p-space; σ-disjoint base; uniform base; pseudocompact; point-countable base; pseudo-ω_1-compact
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