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On computation of minimal free resolutions over solvable polynomial algebras

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Abstract: Let $A = K[a_1, \dots, a_n]$ be a (noncommutative) solvable polynomial algebra over a field K in the sense of A. Kandri-Rody and V. Weispfenning [*Non-commutative Gröbner bases in algebras of solvable type*, J. Symbolic Comput. **9** (1990), 1–26]. This paper presents a comprehensive study on the computation of minimal free resolutions of modules over A in the following two cases: (1) $A = \bigoplus_{p \in \mathbb{N}} A_p$ is an \mathbb{N} -graded algebra with the degree-0 homogeneous part $A_0 = K$; (2) A is an \mathbb{N} -filtered algebra with the filtration $\{F_p A\}_{p \in \mathbb{N}}$ determined by a positive-degree function on A .

Keywords: solvable polynomial algebra; Gröbner basis; minimal free resolution

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