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Semicommutativity of the rings relative to prime radical

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Abstract: In this paper, we introduce a new kind of rings that behave like semicommutative rings, but satisfy yet more known results. This kind of rings is called P-semicommutative. We prove that a ring R is P-semicommutative if and only if $R[x]$ is P-semicommutative if and only if $R[x, x^{-1}]$ is P-semicommutative. Also, if $R[[x]]$ is P-semicommutative, then R is P-semicommutative. The converse holds provided that $P(R)$ is nilpotent and R is power serieswise Armendariz. For each positive integer n, R is P-semicommutative if and only if $T_n(R)$ is P-semicommutative. For a ring R of bounded index 2 and a central nilpotent element s, R is P-semicommutative if and only if $K_s(R)$ is P-semicommutative. If T is the ring of a Morita context $(A, B, M, N, \psi, \varphi)$ with zero pairings, then T is P-semicommutative if and only if A and B are P-semicommutative. Many classes of such rings are constructed as well. We also show that the notions of clean rings and exchange rings coincide for P-semicommutative rings.

Keywords: semicommutative ring; P-semicommutative ring; prime radical of a ring
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