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Diagonals of separately continuous functions of n variables with values in strongly σ-metrizable spaces

Comment.Math.Univ.Carolin. 57,1 (2016) 103 –122.

Abstract: We prove the result on Baire classification of mappings $f : X \times Y \to Z$ which are continuous with respect to the first variable and belongs to a Baire class with respect to the second one, where X is a PP-space, Y is a topological space and Z is a strongly σ-metrizable space with additional properties. We show that for any topological space X, special equiconnected space Z and a mapping $g : X \to Z$ of the $(n-1)$-th Baire class there exists a strongly separately continuous mapping $f : X^n \to Z$ with the diagonal g. For wide classes of spaces X and Z we prove that diagonals of separately continuous mappings $f : X^n \to Z$ are exactly the functions of the $(n-1)$-th Baire class. An example of equiconnected space Z and a Baire-one mapping $g : [0,1] \to Z$, which is not a diagonal of any separately continuous mapping $f : [0,1]^2 \to Z$, is constructed.

Keywords: diagonal of a mapping; separately continuous mapping; Baire-one mapping; equiconnected space; strongly σ-metrizable space
AMS Subject Classification: Primary 54C08, 54C05; Secondary 26B05

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