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On a question of $C_c(X)$

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Abstract: In this short article we answer the question posed in Ghadermazi M., Karamzadeh O.A.S., Namdari M., *On the functionally countable subalgebra of $C(X)$*, Rend. Sem. Mat. Univ. Padova **129** (2013), 47–69. It is shown that $C_c(X)$ is isomorphic to some ring of continuous functions if and only if $\upsilon_0 X$ is functionally countable. For a strongly zero-dimensional space X, this is equivalent to say that X is functionally countable. Hence for every P-space it is equivalent to pseudo-$\aleph_0$-compactness.

Keywords: zero-dimensional space; strongly zero-dimensional space; $\mathbb{N}$-compact space; Banaschewski compactification; character; ring homomorphism; functionally countable subring; functional separability
AMS Subject Classification: Primary 54C30, 54D35, 46E25; Secondary 54D60, 54C40

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