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Remarks on LBI-subalgebras of $C(X)$

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Abstract: Let $A(X)$ denote a subalgebra of $C(X)$ which is closed under local bounded inversion, briefly, an LBI-subalgebra. These subalgebras were first introduced and studied in Redlin L., Watson S., *Structure spaces for rings of continuous functions with applications to realcompactifications*, Fund. Math. **152** (1997), 151–163. By characterizing maximal ideals of $A(X)$, we generalize the notion of z_A^β-ideals, which was first introduced in Acharyya S.K., De D., *An interesting class of ideals in subalgebras of $C(X)$ containing $C^*(X)$*, Comment. Math. Univ. Carolin. **48** (2007), 273–280 for intermediate subalgebras, to the LBI-subalgebras. Using these, it is simply shown that the structure space of every LBI-subalgebra is homeomorphic with a quotient of βX. This gives a different approach to the results of Redlin L., Watson S., *Structure spaces for rings of continuous functions with applications to realcompactifications*, Fund. Math. **152** (1997), 151–163 and also shows that the Banaschewski-compactification of a zero-dimensional space X is a quotient of βX. Finally, we consider the class of complete rings of functions which was first defined in Byun H.L., Redlin L., Watson S., *Local invertibility in subrings of $C^*(X)$*, Bull. Austral. Math. Soc. **46**(1992), 449–458. Showing that every such subring is an LBI-subalgebra, we prove that the compactification of X associated to each complete ring of functions, which is identified in Byun H.L., Redlin L., Watson S., *Local invertibility in subrings of $C^*(X)$*, Bull. Austral. Math. Soc. **46**(1992), 449–458 via the mapping $\mathcal{Z}_A$, is in fact, the structure space of that subring. Henceforth, some statements in Byun H.L., Redlin L., Watson S., *Local invertibility in subrings of $C^*(X)$*, Bull. Austral. Math. Soc. **46**(1992), 449–458 could be proved in a different way.

Keywords: local bounded inversion; structure space; z_A^β-ideal; complete ring of functions
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