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A weighted inequality for the Hardy operator involving suprema

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Abstract: Let u be a weight on $(0,\infty)$. Assume that u is continuous on $(0,\infty)$. Let the operator S_u be given at measurable non-negative function φ on $(0,\infty)$ by

$$S_u\varphi(t) = \sup_{0<\tau\le t} u(\tau)\varphi(\tau).$$

We characterize weights v, w on $(0,\infty)$ for which there exists a positive constant C such that the inequality

$$\left(\int_0^\infty [S_u\varphi(t)]^q w(t)\,dt\right)^{\frac{1}{q}} \lesssim \left(\int_0^\infty [\varphi(t)]^p v(t)\,dt\right)^{\frac{1}{p}}$$

holds for every $0 < p, q < \infty$. Such inequalities have been used in the study of optimal Sobolev embeddings and boundedness of certain operators on classical Lorenz spaces.

Keywords: Hardy operators involving suprema; weighted inequalities
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