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Functionally countable subalgebras and some properties of the Banaschewski compactification

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Abstract: Let X be a zero-dimensional space and $C_c(X)$ be the set of all continuous real valued functions on X with countable image. In this article we denote by $C_c^K(X)$ (resp., $C_c^\psi(X)$) the set of all functions in $C_c(X)$ with compact (resp., pseudocompact) support. First, we observe that $C_c^K(X) = O_c^{\beta_0 X\setminus X}$ (resp., $C_c^\psi(X) = M_c^{\beta_0 X\setminus \upsilon_0 X}$), where $\beta_0 X$ is the Banaschewski compactification of X and $\upsilon_0 X$ is the $\mathbb{N}$-compactification of X. This implies that for an $\mathbb{N}$-compact space X, the intersection of all free maximal ideals in $C_c(X)$ is equal to $C_c^K(X)$, i.e., $M_c^{\beta_0 X\setminus X} = C_c^K(X)$. By applying methods of functionally countable subalgebras, we then obtain some results in the remainder of the Banaschewski compactification. We show that for a non-pseudocompact zero-dimensional space X, the set $\beta_0 X\setminus \upsilon_0 X$ has cardinality at least $2^{2^{\aleph_0}}$. Moreover, for a locally compact and $\mathbb{N}$-compact space X, the remainder $\beta_0 X \setminus X$ is an almost P-space. These results lead us to find a class of Parovičenko spaces in the Banaschewski compactification of a non pseudocompact zero-dimensional space. We conclude with a theorem which gives a lower bound for the cellularity of the subspaces $\beta_0 X \setminus \upsilon_0 X$ and $\beta_0 X \setminus X$, whenever X is a zero-dimensional, locally compact space which is not pseudocompact.

Keywords: zero-dimensional space; strongly zero-dimensional space; $\mathbb{N}$-compact space; Banaschewski compactification; pseudocompact space; functionally countable subalgebra; support; cellularity; remainder; almost P-space; Parovičenko space

AMS Subject Classification: Primary 54C30, 54C40, 54A25; Secondary 54D40, 54D60, 54G05

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