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On dicyclic groups as inner mapping groups of finite loops

Abstract: Let G be a finite group with a dicyclic subgroup H. We show that if there exist H-connected transversals in G, then G is a solvable group. We apply this result to loop theory and show that if the inner mapping group $I(Q)$ of a finite loop Q is dicyclic, then Q is a solvable loop. We also discuss a more general solvability criterion in the case where $I(Q)$ is a certain type of a direct product.

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