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On Hattori spaces

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Abstract: For a subset A of the real line $\mathbb{R}$, Hattori space $H(A)$ is a topological space whose underlying point set is the reals $\mathbb{R}$ and whose topology is defined as follows: points from A are given the usual Euclidean neighborhoods while remaining points are given the neighborhoods of the Sorgenfrey line. In this paper, among other things, we give conditions on A which are sufficient and necessary for $H(A)$ to be respectively almost Čech-complete, Čech-complete, quasicomplete, Čech-analytic and weakly separated (in Tkacenko sense). Some of these results solve questions raised by V.A. Chatyrko and Y. Hattori.

Keywords: Hattori space; Čech-complete space; Čech-analytic space; neighborhood assignment; Sorgenfrey line; scattered set; weakly separated space
AMS Subject Classification: 54C05, 54C35, 54C45, 54C99

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