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Parabolicity and rigidity of spacelike hypersurfaces immersed in a Lorentzian Killing warped product

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Abstract: In this paper, we extend a technique due to Romero et al. establishing sufficient conditions to guarantee the parabolicity of complete spacelike hypersurfaces immersed into a Lorentzian Killing warped product whose Riemannian base has parabolic universal Riemannian covering. As applications, we obtain rigidity results concerning these hypersurfaces. A particular study of entire Killing graphs is also made.

Keywords: Lorentzian Killing warped product; complete spacelike hypersurfaces; parabolic spacelike hypersurfaces; entire Killing graphs

AMS Subject Classification: Primary 53C42; Secondary 53B30, 53C50

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