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Semifields and a theorem of Abhyankar

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Abstract: Abhyankar proved that every field of finite transcendence degree over \mathbb{Q} or over a finite field is a homomorphic image of a subring of the ring of polynomials $\mathbb{Z}[T_1, \dots, T_n]$ (for some n depending on the field). We conjecture that his result cannot be substantially strengthened and show that our conjecture implies a well-known conjecture on the additive idempotence of semifields that are finitely generated as semirings.

Keywords: Abhyankar’s construction; semiring; semifield; finitely generated; additively idempotent

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