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On star covering properties related to countable compactness and pseudocompactness

Abstract: We prove a number of results on star covering properties which may be regarded as either generalizations or specializations of topological properties related to the ones mentioned in the title of the paper. For instance, we give a new, entirely combinatorial proof of the fact that Ψ-spaces constructed from infinite almost disjoint families are not star-compact. By going a little further we conclude that if X is a star-compact space within a certain class, then X is neither first countable nor separable. We also show that if a topological space is pseudonormal and has countable extent, then its Alexandroff duplicate satisfies property (a). A number of problems and questions are also presented.

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