# Pavel Růžička
*The graphs of join-semilattices and the shape of congruence lattices of particle lattices*

**Abstract:** We attach to each $\langle 0, \vee \rangle$-semilattice $\boldsymbol{S}$ a graph $\boldsymbol{G_S}$ whose vertices are join-irreducible elements of $\boldsymbol{S}$ and whose edges correspond to the reflexive dependency relation. We study properties of the graph $\boldsymbol{G_S}$ both when $\boldsymbol{S}$ is a join-semilattice and when it is a lattice. We call a $\langle 0, \vee \rangle$-semilattice $\boldsymbol{S}$ *particle* provided that the set of its join-irreducible elements satisfies DCC and join-generates $\boldsymbol{S}$. We prove that the congruence lattice of a particle lattice is anti-isomorphic to the lattice of all hereditary subsets of the corresponding graph that are closed in a certain zero-dimensional topology. Thus we extend the result known for principally chain finite lattices.

## References

[1] Day A., *Characterizations of finite lattices that are bounded-homomorphic images or sublattices of free lattices*, Canad. J. Math. **31** (1979), 69–78.

[2] Engelking R., *General Topology*, Heldermann, Berlin, 1989.

[3] Freese R., Ježek J., Nation J.B., *Free Lattices*, Mathematical Surveys and Monographs, 42, American Mathematical Society, Providence, Rhode Island, 1995.

[4] Grätzer G., *General Lattice Theory*, second edition, Birkhäuser, Basel, 1998.

[5] Grätzer G., Schmidt E.T., *On congruence lattices of lattices*, Acta Math. Acad. Sci. Hungar. **13** (1962), 179–185.

[6] König D., *Über eine Schlussweise aus dem Endlichen ins Unendliche*, Acta Sci. Math. (Szeged) **3** (1927), 121–130.

[7] Nation J.B., *Notes on lattice theory*, available online.

[8] Pudlák P., Tůma J., *Yeast graphs and fermentation of lattices*, Lattice Theory (Proc. Colloq., Szeged, 1974), Colloq. Math. Soc. János Bolyai, vol. 14, North-Holland, Amsterdam, 1976, pp. 301–341.

[9] Rhodes J.B., *Modular and distributive semilattices*, Trans. Amer. Math. Soc. **201** (1975), 31–41.

[10] Tischendorf M., *The representation problem for algebraic distributive lattices*, Ph.D. Thesis, TH Darmstadt, 1992.

[11] Wehrung F., *Representation of algebraic distributive lattices with $\aleph_1$ compact elements as ideal lattices of regular rings*, Publ. Mat. (Barcelona) **44** (2000), 419–435.