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Some applications of the point-open subbase game

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Abstract: Given a subbase $\mathcal{S}$ of a space X, the game $PO(\mathcal{S}, X)$ is defined for two players P and O who respectively pick, at the n-th move, a point $x_n \in X$ and a set $U_n \in \mathcal{S}$ such that $x_n \in U_n$. The game stops after the moves $\{x_n, U_n : n \in \o\}$ have been made and the player P wins if $\bigcup_{n\in\o} U_n = X$; otherwise O is the winner. Since $PO(\mathcal{S}, X)$ is an evident modification of the well-known point-open game $PO(X)$, the primary line of research is to describe the relationship between $PO(X)$ and $PO(\mathcal{S}, X)$ for a given subbase $\mathcal{S}$. It turns out that, for any subbase $\mathcal{S}$, the player P has a winning strategy in $PO(\mathcal{S}, X)$ if and only if he has one in $PO(X)$. However, these games are not equivalent for the player O: there exists even a discrete space X with a subbase $\mathcal{S}$ such that neither P nor O has a winning strategy in the game $PO(\mathcal{S}, X)$. Given a compact space X, we show that the games $PO(\mathcal{S}, X)$ and $PO(X)$ are equivalent for any subbase $\mathcal{S}$ of the space X.

Keywords: point-open game; subbase; winning strategy; players; discrete space; compact space; scattered space; measurable cardinal
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