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*On a class of abstract degenerate fractional differential equations of parabolic type*

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**Abstract:** In this paper, we investigate a class of abstract degenerate fractional differential equations with Caputo derivatives. We consider subordinated fractional resolvent families generated by multivalued linear operators, which do not have removable singularities at the origin. Semi-linear degenerate fractional Cauchy problems are also considered in this context.

**Keywords:** abstract degenerate fractional differential equations; infinitely differentiable fractional resolvent families; multivalued linear operators; semi-linear degenerate fractional Cauchy problems; Caputo fractional derivatives

**AMS Subject Classification:** 47D03, 47D06, 47D62, 47D99, 47G20

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