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A note on Dunford-Pettis like properties and complemented spaces of operators

Abstract: Equivalent formulations of the Dunford-Pettis property of order p (DPP$_p$), $1 < p < \infty$, are studied. Let L(X, Y), W(X, Y), K(X, Y), U(X, Y), and C$_p(X, Y)$ denote respectively the sets of all bounded linear, weakly compact, compact, unconditionally converging, and p-convergent operators from X to Y. Classical results of Kalton are used to study the complementability of the spaces W(X, Y) and K(X, Y) in the space C$_p(X, Y)$, and of C$_p(X, Y)$ in U(X, Y) and L(X, Y).

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