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A nice subclass of functionally countable spaces

Abstract: A space X is functionally countable if $f(X)$ is countable for any continuous function $f\colon X \to \mathbb{R}$. We will call a space X exponentially separable if for any countable family $\mathcal{F}$ of closed subsets of X, there exists a countable set $A \subset X$ such that $A \cap \bigcap \mathcal{G} \neq \emptyset$ whenever $\mathcal{G} \subset \mathcal{F}$ and $\bigcap \mathcal{G} \neq \emptyset$. Every exponentially separable space is functionally countable; we will show that for some nice classes of spaces exponential separability coincides with functional countability. We will also establish that the class of exponentially separable spaces has nice categorical properties: it is preserved by closed subspaces, countable unions and continuous images. Besides, it contains all Lindelöf P-spaces as well as some wide classes of scattered spaces. In particular, if a scattered space is either Lindelöf or ω-bounded, then it is exponentially separable.

Keywords: countably compact space; Lindelöf space; Lindelöf P-space; functionally countable space; exponentially separable space; retraction; scattered space; extent; Sokolov space; weakly Sokolov space; function space
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