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Coloring Cantor sets and resolvability of pseudocompact spaces

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Abstract: Let us denote by $\Phi(\lambda, \mu)$ the statement that $\mathbb{B}(\lambda) = D(\lambda)^\omega$, i.e. the Baire space of weight λ, has a coloring with μ colors such that every homeomorphic copy of the Cantor set $\mathbb{C}$ in $\mathbb{B}(\lambda)$ picks up all the μ colors. We call a space X π-regular if it is Hausdorff and for every nonempty open set U in X there is a nonempty open set V such that $\overline{V} \subset U$. We recall that a space X is called feebly compact if every locally finite collection of open sets in X is finite. A Tychonov space is pseudocompact if and only if it is feebly compact. The main result of this paper is the following: Let X be a crowded feebly compact π-regular space and μ be a fixed (finite or infinite) cardinal. If $\Phi(\lambda, \mu)$ holds for all $\lambda < \hat{c}(X)$ then X is μ-resolvable, i.e. X contains μ pairwise disjoint dense subsets. (Here $\hat{c}(X)$ is the smallest cardinal κ such that X does not contain κ many pairwise disjoint open sets.) This significantly improves earlier results of [van Mill J., *Every crowded pseudocompact ccc space is resolvable*, Topology Appl. 213 (2016), 127–134], or [Ortiz-Castillo Y. F., Tomita A. H., *Crowded pseudocompact Tychonoff spaces of cellularity at most the continuum are resolvable*, Conf. talk at Toposym 2016].

Keywords: pseudocompact; feebly compact; resolvable; Baire space; coloring; Cantor set
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