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*Fraïssé structures and a conjecture of Furstenberg*

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**Abstract:** We study problems concerning the Samuel compactification of the automorphism group of a countable first-order structure. A key motivating question is a problem of Furstenberg and a counter-conjecture by Pestov regarding the difference between  $S(G)$ , the Samuel compactification, and  $E(M(G))$ , the enveloping semigroup of the universal minimal flow. We resolve Furstenberg’s problem for several automorphism groups and give a detailed study in the case of  $G = S_\infty$ , leading us to define and investigate several new types of ultrafilters on a countable set.

**Keywords:** Fraïssé structures; enveloping semigroups; universal minimal flow

**AMS Subject Classification:** 37B05, 05C63, 03E05, 22F50

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