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Ultrafilter extensions of asymptotic density

Abstract: We characterize for which ultrafilters on ω is the ultrafilter extension of the asymptotic density on natural numbers σ-additive on the quotient boolean algebra $\mathcal{P}(\omega)/d_{\mathcal{U}}$ or satisfies similar additive condition on $\mathcal{P}(\omega)/\mathrm{fin}$. These notions were defined in [Blass A., Frankiewicz R., Plebanek G., Ryll-Nardzewski C., *A Note on extensions of asymptotic density*, Proc. Amer. Math. Soc. **129** (2001), no. 11, 3313–3320] under the name $\boldsymbol{AP}$(null) and $\boldsymbol{AP}$(*). We also present a characterization of a P- and semiselective ultrafilters using the ultraproduct of σ-additive measures.

References

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