

**Jan Hubička, Matěj Konečný, Jaroslav Nešetřil**  
*A combinatorial proof of the extension property for partial isometries*

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**Abstract:** We present a short and self-contained proof of the extension property for partial isometries of the class of all finite metric spaces.

**Keywords:** metric space; Hrushovski property; extension property for partial automorphisms; homogeneous structure; amalgamation class

**AMS Subject Classification:** 20B27, 05E18, 54E35, 20F05, 22F50, 37B05

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