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*Local-global convergence,  
an analytic and structural approach*

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**Abstract:** Based on methods of structural convergence we provide a unifying view of local-global convergence, fitting to model theory and analysis. The general approach outlined here provides a possibility to extend the theory of local-global convergence to graphs with unbounded degrees. As an application, we extend previous results on continuous clustering of local convergent sequences and prove the existence of modeling quasi-limits for local-global convergent sequences of nowhere dense graphs.

**Keywords:** structural limit; Borel structure; modeling; local-global convergence

**AMS Subject Classification:** 03C13, 03C98, 05C99

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