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*Regularity problem for one class of nonlinear parabolic systems with non-smooth in time principal matrices*

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**Abstract:** Partial regularity of solutions to a class of second order nonlinear parabolic systems with non-smooth in time principal matrices is proved in the paper. The coefficients are assumed to be measurable and bounded in the time variable and VMO-smooth in the space variables uniformly with respect to time. To prove the result, we apply the so-called  $A(t)$ -caloric approximation method. The method was applied by the authors earlier to study regularity of quasilinear systems.

**Keywords:** nonlinear parabolic systems; regularity problem

**AMS Subject Classification:** 35B65, 35D30, 35K99

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