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Results of nonexistence of solutions for some nonlinear evolution problems

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Abstract: In the present paper, we prove nonexistence results for the following nonlinear evolution equation, see works of T. Cazenave and A. Haraux (1990) and S. Zheng (2004),

$$u_{tt} + f(x)u_t + (-\Delta)^{\alpha/2}(u^m) = h(t,x)|u|^p,$$

posed in $(0,T)\times\mathbb{R}^N$, where $(-\Delta)^{\alpha/2}$, $0<\alpha\leq 2$ is $\alpha/2$-fractional power of $-\Delta$. Our method of proof is based on suitable choices of the test functions in the weak formulation of the sought solutions. Then, we extend this result to the case of a 2×2 system of the same type.

Keywords: nonexistence; test functions; global weak solution; fractional Laplacian; critical exponent
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