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A remark on functions continuous on all lines

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Abstract: We prove that each linearly continuous function f on $\mathbb{R}^n$ (i.e., each function continuous on all lines) belongs to the first Baire class, which answers a problem formulated by K. C. Ciesielski and D. Miller (2016). The same result holds also for f on an arbitrary Banach space X, if f has moreover the Baire property. We also prove (extending a known finite-dimensional result) that such f on a separable X is continuous at all points outside a first category set which is also null in any usual sense.

Keywords: linear continuity; Baire class one; discontinuity set; Banach space
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