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On the integral representation of finely superharmonic functions

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Abstract: In the present paper we study the integral representation of nonnegative finely superharmonic functions in a fine domain subset U of a Brelot $\mathcal{P}$-harmonic space Ω with countable base of open subsets and satisfying the axiom D. When Ω satisfies the hypothesis of uniqueness, we define the Martin boundary of U and the Martin kernel K and we obtain the integral representation of invariant functions by using the kernel K. As an application of the integral representation we extend to the cone $\mathcal{S}(\mathcal{U})$ of nonnegative finely superharmonic functions in U a partition theorem of Brelot. We also establish an approximation result of invariant functions by finely harmonic functions in the case where the minimal invariant functions are finely harmonic.

Keywords: finely harmonic function; finely superharmonic function; fine potential; fine Green kernel; integral representation; Martin boundary; fine Riesz-Martin kernel
AMS Subject Classification: 31D05, 31C35, 31C40

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