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A note on generalizations of semisimple modules

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Abstract: A left module M over an arbitrary ring is called an $\mathcal{RD}$-module (or an $\mathcal{RS}$-module) if every submodule N of M with $\mathrm{Rad}(M) \subseteq N$ is a direct summand of (a supplement in, respectively) M. In this paper, we investigate the various properties of $\mathcal{RD}$-modules and $\mathcal{RS}$-modules. We prove that M is an $\mathcal{RD}$-module if and only if $M = \mathrm{Rad}(M) \oplus X$, where X is semisimple. We show that a finitely generated $\mathcal{RS}$-module is semisimple. This gives us the characterization of semisimple rings in terms of $\mathcal{RS}$-modules. We completely determine the structure of these modules over Dedekind domains.

Keywords: radical; supplement
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