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Countable compactness of lexicographic products of GO-spaces

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Abstract: We characterize the countable compactness of lexicographic products of GO-spaces. Applying this characterization about lexicographic products, we see:

- the lexicographic product X^2 of a countably compact GO-space X need not be countably compact,
- ω_1^2 , $\omega_1 \times \omega$, $(\omega + 1) \times (\omega_1 + 1) \times \omega_1 \times \omega$, $\omega_1 \times \omega \times \omega_1$, $\omega_1 \times \omega \times \omega_1 \times \omega \times \cdots$, $\omega_1 \times \omega^\omega$, $\omega_1 \times \omega^\omega \times (\omega + 1)$, ω_1^ω , $\omega_1^\omega \times (\omega_1 + 1)$ and $\prod_{n \in \omega} \omega_{n+1}$ are countably compact,
- $\omega \times \omega_1$, $(\omega + 1) \times (\omega_1 + 1) \times \omega \times \omega_1$, $\omega \times \omega_1 \times \omega \times \omega_1 \times \cdots$, $\omega \times \omega_1^\omega$, $\omega_1 \times \omega^\omega \times \omega_1$, $\omega_1^\omega \times \omega$, $\prod_{n \in \omega} \omega_n$ and $\prod_{n \leq \omega} \omega_{n+1}$ are not countably compact,
- $[0, 1]_{\mathbb{R}} \times \omega_1$, where $[0, 1]_{\mathbb{R}}$ denotes the half open interval in the real line \mathbb{R} , is not countably compact,
- $\omega_1 \times [0, 1]_{\mathbb{R}}$ is countably compact,
- both $\mathbb{S} \times \omega_1$ and $\omega_1 \times \mathbb{S}$ are not countably compact,
- $\omega_1 \times (-\omega_1)$ is not countably compact, where for a GO-space $X = \langle X, <_X, \tau_X \rangle$, $-X$ denotes the GO-space $\langle X, >_X, \tau_X \rangle$.

Keywords: lexicographic product; GO-space; LOTS; countably compact product

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