

Rabah Khemis, Abdelouaheb Ardjouni, Ahlème Bouakkaz, Ahcene Djoudi

Periodic solutions of a class of third-order differential equations with two delays depending on time and state

Comment.Math.Univ.Carolin. 60,3 (2019) 379 –399.

Abstract: The goal of the present paper is to establish some new results on the existence, uniqueness and stability of periodic solutions for a class of third order functional differential equations with state and time-varying delays. By Krasnoselskii's fixed point theorem, we prove the existence of periodic solutions and under certain sufficient conditions, the Banach contraction principle ensures the uniqueness of this solution. The results obtained in this paper are illustrated by an example.

Keywords: periodic solution; iterative differential equation; fixed point theorem; Green's function

AMS Subject Classification: 39B12, 39B82

REFERENCES

- [1] Babbage C., *An essay towards the calculus of functions*, Philosophical Transactions of The Royal Society of London **105** (1815), 389–423.
- [2] Berinde V., *Existence and approximation of solutions of some first order iterative differential equations*, Miskolc Math. Notes **11** (2010), no. 1, 13–26.
- [3] Cooke K. L., *Functional-differential equations: Some models and perturbation problems*, Differential Equations and Dynamical Systems, Proc. Internat. Sympos., Mayaguez, 1965, Academic Press, New York, 1967, pages 167–183.
- [4] Driver R. D., *Delay-differential Equations and an Application to a Two-body Problem of Classical Electrodynamics*, Thesis Ph.D., University of Minnesota, 1960.
- [5] Eder E., *The functional-differential equation $x'(t) = x(x(t))$* , J. Differential Equations **54** (1984), no. 3, 390–400.
- [6] Fečkan M., *On a certain type of functional-differential equations*, Math. Slovaca **43** (1993), no. 1, 39–43.
- [7] Ge W., Mo Y., *Existence of solutions to differential-iterative equation*, J. Beijing Inst. Tech. **6** (1997), no. 3, 192–200.
- [8] Luran M., *Existence results for some differential equations with deviating argument*, Filomat **25** (2011), no. 2, 21–31.
- [9] Li Y., Kuang Y., *Periodic solutions in periodic state-dependent delay equations and population models*, Proc. Amer. Math. Soc. **130** (2002), no. 5, 1345–1353.
- [10] Pelczyr A., *On some iterative differential equations. I*, Zeszyty Nauk. Uniw. Jagiello. Prace Matemat. No. **12** (1968), 53–56.
- [11] Ren J., Siegmund S., Chen Y., *Positive periodic solutions for third-order nonlinear differential equations*, Electron. J. Differential Equations (2011), No. 66, 19 pages.
- [12] Smart D. R., *Fixed Point Theorems*, Cambridge Tracts in Mathematics, 66, Cambridge University Press, London, 1974.
- [13] Wang K., *On the equation $x'(t) = f(x(x(t)))$* , Funkcial. Ekvac. **33** (1990), no. 3, 405–425.
- [14] Zhao H. Y., Liu J., *Periodic solutions of an iterative functional differential equation with variable coefficients*, Math. Methods Appl. Sci. **40** (2017), no. 1, 286–292.
- [15] Zhao H. Y., Fečkan M., *Periodic solutions for a class of differential equations with delays depending on state*, Math. Commun. **23** (2018), no. 1, 29–42.