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### *On commutative rings whose maximal ideals are idempotent*

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**Abstract:** We prove that for a commutative ring  $R$ , every noetherian (artinian)  $R$ -module is quasi-injective if and only if every noetherian (artinian)  $R$ -module is quasi-projective if and only if the class of noetherian (artinian)  $R$ -modules is socle-fine if and only if the class of noetherian (artinian)  $R$ -modules is radical-fine if and only if every maximal ideal of  $R$  is idempotent.

**Keywords:** artinian module; modules of finite length; noetherian module; quasi-injective module; quasi-projective module; radical-fine class of modules; socle-fine class of modules

**AMS Subject Classification:** 13C13, 13E05, 13E10, 13E99

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