

Jiří Adámek, Andrew D. Brooke-Taylor, Tim Champion, Leonid Positselski, Jiří Rosický
Colimit-dense subcategories

Comment.Math.Univ.Carolin. 60,4 (2019) 447–462.

Abstract: Among cocomplete categories, the locally presentable ones can be defined as those with a strong generator consisting of presentable objects. Assuming Vopěnka’s Principle, we prove that a cocomplete category is locally presentable if and only if it has a colimit dense subcategory and a generator consisting of presentable objects. We further show that a 3-element set is colimit-dense in \mathbf{Set}^{op} , and spaces of countable dimension are colimit-dense in \mathbf{Vec}^{op} .

Keywords: locally presentable category; colimit-dense subcategory; Vopěnka’s Principle

AMS Subject Classification: 18C35, 18A30, 03E55

REFERENCES

- [1] Adámek J., Herrlich H., Reiterman J., *Cocompleteness almost implies completeness*, Proc. Conf. Categorical Topology and Its Relation to Analysis, Algebra and Combinatorics, Prague, 1988, World Sci. Publ., Teaneck (1989), pages 246–256.
- [2] Adámek J., Rosický J., *Locally Presentable and Accessible Categories*, London Mathematical Society Lecture Note Series, 189, Cambridge University Press, Cambridge, 1994.
- [3] Bardavid C., *Profinite completion and double-dual: isomorphisms and counter-examples*, available at arXiv:0801.2955v1 [math.GR] (2008), 8 pages.
- [4] Benson D. J., *Infinite dimensional modules for finite groups*, Infinite Length Modules, Bielefeld, 1998, Trends Math., Birkhäuser, Basel, 2000, pages 251–272.
- [5] Börger R., *Coproducts and ultrafilters*, J. Pure Appl. Algebra **46** (1987), no. 1, 35–47.
- [6] Galvin F., Horn A., *Operations preserving all equivalence relations*, Proc. Amer. Math. Soc. **24** (1970), 521–523.
- [7] Isbell J. R., *Adequate subcategories*, Illinois J. Math. **4** (1960), 541–552.
- [8] Isbell J. R., *Subobjects, adequacy, completeness and categories of algebras*, Rozprawy Mat. **36** (1964), 33 pages.
- [9] Kennison J. F., Gildenhuys D., *Equational completion, model induced triples and pro-objects*, J. Pure Appl. Algebra **1** (1971), no. 4, 317–346.
- [10] Leinster T., *Codensity and the ultrafilter monad*, Theory Appl. Categ. **28** (2013), no. 13, 332–370.
- [11] Rosický J., *Codensity and binding categories*, Comment. Math. Univ. Carolinae **16** (1975), no. 3, 515–529.
- [12] Rosický J., Trnková V., Adámek J., *Unexpected properties of locally presentable categories*, Algebra Universalis **27** (1990), no. 2, 153–170.