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Contractible simplicial objects

Comment.Math.Univ.Carolin. 60,4 (2019) 473–495.

Abstract: We raise the question of when a simplicial object in a category is deemed contractible. The literature offers three definitions. One is the existence of an “extra degeneracy”, indexed by -1 , which does not quite live up to the name. This can be strengthened to a “strong extra degeneracy”. Another possibility is that it be homotopic to a constant simplicial object. Despite claims in the literature to the contrary, we show that all three are distinct concepts with strong extra degeneracy implies extra degeneracy implies homotopic to a constant and give explicit examples to show the converses fail.

Keywords: contractible; homotopic to a constant; reduced homotopy; partial simplicial object

AMS Subject Classification: 18G30, 55U10

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