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Split extensions and semidirect products of unitary magmas

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Abstract: We develop a theory of split extensions of unitary magmas, which includes defining such extensions and describing them via suitably defined semidirect product, yielding an equivalence between the categories of split extensions and of (suitably defined) actions of unitary magmas on unitary magmas. The class of split extensions is pullback stable but not closed under composition. We introduce two subclasses of it that have both of these properties.

Keywords: unitary magma; split extension; firm split extension; semidirect product

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