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*Axiom  $T_D$  and the Simmons sublocale theorem*

Comment.Math.Univ.Carolin. 60,4 (2019) 541–551.

**Abstract:** More precisely, we are analyzing some of H. Simmons, S. B. Niefield and K. I. Rosenthal results concerning sublocales induced by subspaces. H. Simmons was concerned with the question when the coframe of sublocales is Boolean; he recognized the role of the axiom  $T_D$  for the relation of certain degrees of scatteredness but did not emphasize its role in the relation between sublocales and subspaces. S. B. Niefield and K. I. Rosenthal just mention this axiom in a remark about Simmons' result. In this paper we show that the role of  $T_D$  in this question is crucial. Concentration on the properties of  $T_D$ -spaces and technique of sublocales in this context allows us to present a simple, transparent and choice-free proof of the scatteredness theorem.

**Keywords:** frame; locale; sublocale; coframe of sublocales; spatial sublocale; induced sublocale;  $T_D$ -separation; covered prime element; scattered space; weakly scattered space  
**AMS Subject Classification:** 06D22, 54D10

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