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Order-enriched solid functors

Comment.Math.Univ.Carolin. 60,4 (2019) 553 –580.

Abstract: Order-enriched solid functors, as presented in this paper in two versions, enjoy many of the strong properties of their ordinary counterparts, including the transfer of the existence of weighted (co)limits from their codomains to their domains. The ordinary version of the notion first appeared in Trnková's work on automata theory of the 1970s and was subsequently studied by others under various names, before being put into a general enriched context by C. Anghel. Our focus in this paper is on differentiating the order-enriched notion from the ordinary one, mostly in terms of the functor's behaviour with respect to specific weighted (co)limits, and on the presentation of examples, which include functors of general varieties of ordered algebras and special ones, such as ordered vector spaces.

Keywords: ordered category; (strongly) order-solid functor; weighted (co)limit; ordered algebra
AMS Subject Classification: 18A22, 18A30, 18B35, 06F99

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