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A coalgebraic view on reachability

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Abstract: Coalgebras for an endofunctor provide a category theoretic framework for modeling a wide range of state-based systems of various types. We provide an iterative construction of the reachable part of a given pointed coalgebra that is inspired by and resembles the standard breadth-first search procedure to compute the reachable part of a graph. We also study coalgebras in Kleisli categories: for a functor extending a functor on the base category, we show that the reachable part of a given pointed coalgebra can be computed in that base category.

Keywords: coalgebra; reachability; Kleisli category

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