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Weingarten hypersurfaces of the spherical type in Euclidean spaces

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Abstract: We generalize a parametrization obtained by A. V. Corro in (2006) in the three-dimensional Euclidean space. Using this parametrization we study a class of oriented hypersurfaces M^n, $n \geq 2$, in Euclidean space satisfying a relation $\sum_{r=1}^{n}(-1)^{r+1} r f^{r-1}\binom{n}{r}H_r = 0$, where H_r is the rth mean curvature and $f \in C^\infty(M^n;\mathbb{R})$, these hypersurfaces are called Weingarten hypersurfaces of the spherical type. This class of hypersurfaces includes the surfaces of the spherical type (Laguerré minimal surfaces). We characterize these hypersurfaces in terms of harmonic applications. Also, we classify the Weingarten hypersurfaces of the spherical type of rotation and we give explicit examples.

Keywords: Weingarten hypersurface; Laguerre minimal surface; rth mean curvature; Laplace–Beltrami operator
AMS Subject Classification: 53C42, 53A35

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