Waldo Arriagada, Jorge Huentutripay
Asymptotic properties of a φ-Laplacian and Rayleigh quotient

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Abstract: In this paper we consider the φ-Laplacian problem with Dirichlet boundary condition,

$$-\mathrm{div}\Big(\varphi(|\nabla u|)\frac{\nabla u}{|\nabla u|}\Big) = \lambda g(\cdot)\varphi(u) \qquad \text{in } \ \Omega, \ \lambda \in \mathbb{R} \ \text{ and } \ u|_{\partial\Omega} = 0.$$

The term φ is a real odd and increasing homeomorphism, g is a nonnegative function in $L^\infty(\Omega)$ and $\Omega \subseteq \mathbb{R}^N$ is a bounded domain. In these notes an analysis of the asymptotic behavior of sequences of eigenvalues of the differential equation is provided. We assume conditions which guarantee the existence of stationary solutions of the system. Under these rather stringent hypotheses we prove that any extremal is both a minimizer and an eigenfunction of the φ-Laplacian. It turns out that if, in addition, a suitable Δ_2-condition holds then any number greater than or equal to the minimum of the Rayleigh quotient is an eigenvalue of the differential equation.

Keywords: Orlicz–Sobolev space; φ-Laplacian; eigenvalue; Rayleigh quotient
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